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Analytical Mechanics Phys (252)

Lecture 9

Examples

The Concept of Potential Energy

Free Fall

***EXAMPLE* (2.3.1)**

The motion of a *freely falling* body is an example of conservative motion. In this case:

Then

$$F = -\frac{dV}{dx} = -mg$$

$$V = mgx + C$$

We can choose $C = 0$, which means that $V = 0$ when $x = 0$.

The **gravitational potential energy** is then $\boxed{V = mgx}$

For instance, let the body be projected upward with initial speed v_0 from the origin $x=0$. These values give;

$$T + V(x) = T_0 + V(x_0) \equiv E$$

$$\tfrac{1}{2} mv^2 + mgx = \tfrac{1}{2} mv_0^2 + 0$$

so;

$$\boxed{v^2 = v_0^2 - 2gx}$$

The turning point of the motion, which is in this case *the maximum height*, is given by setting $v = 0$.
This gives

$$h = x_{\max} = \frac{v_0^2}{2g}$$

Other Solution

Using;

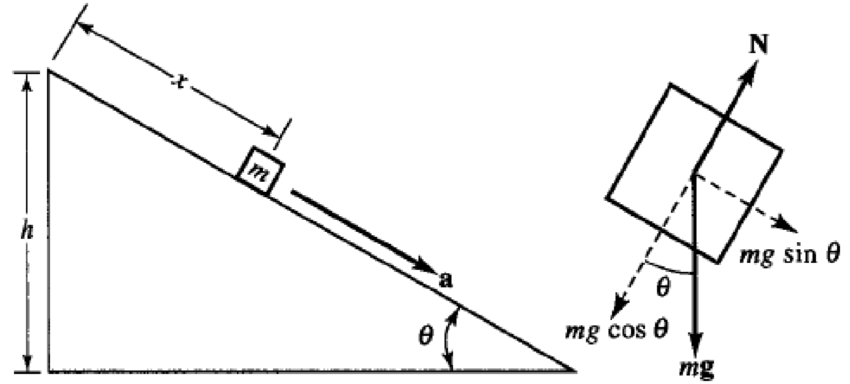
$$2a(x - x_0) = v^2 - v_0^{\,2}$$

we obtain;

$$h = x - x_0 = \frac{-v_0^2}{-2g} = \frac{v_0^2}{2g}$$

EXAMPLE 2.2.1

Consider a block that is free to slide down a smooth, frictionless plane that is inclined at an angle θ to the horizontal. If the height of the plane is h and the block is released from rest at the top ($v_0=0$), what will be its speed when it reaches the bottom?



$$\ddot{x} = a = \frac{F_g}{m} = g \sin\theta$$

and $$x - x_0 = \frac{h}{\sin\theta}$$

Using;

$$2a(x-x_0) = v^2 - v_0^2$$

we obtain;

$$\boxed{v^2 = 2g\sin\theta\left(\frac{h}{\sin\theta}\right) = 2gh}$$

Other Solution

$$T + V(x) = T_0 + V(x_0) \equiv E$$

$$\tfrac{1}{2}mv^2 + 0 = 0 + mgh$$

so; $$\boxed{v^2 = 2gh}$$

Morse Function

EXAMPLE (2.3.3):

The potential energy of a vibrating diatomic molecule as a function of x is given by;

$$V(x) = V_0\left[1 - e^{-(x-x_0)/\delta}\right]^2 - V_0$$

Show that:

1- x_0 is the separation of the two atoms at equilibrium, i.e. when the *potential energy function* is **minimum**.

2- and that $V(x_0) = -V_0$.

Solution

$V(x)$ is min when its derivative (w.r.t) x is zero;

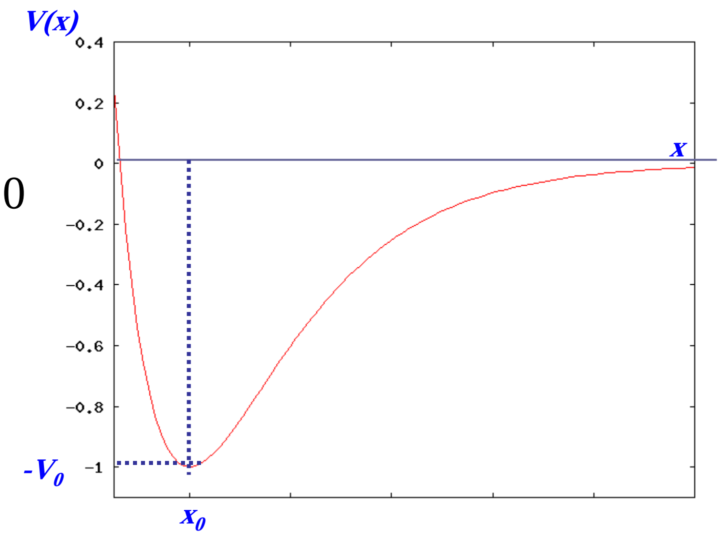
$$F(x) = -\frac{dV(x)}{dx} = 0$$

$$2\frac{V_0}{\delta}\left(1 - e^{-(x-x_0)/\delta}\right)\left(e^{-(x-x_0)/\delta}\right) = 0$$

$$1 - e^{-(x-x_0)/\delta} = 0$$

$$\ln(1) = -(x - x_0)/\delta$$

$$\therefore \quad x = x_0$$



Substituting in the main equation, the value of the min $V(x)$ can be found as;

$$V(x_0) = -V_0$$